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16. Abstract

A description is given of results of research on the motion of sand in a wind. Equations are derived for determining the sand motion. The relationship between wind intensity and amount of moving sand is determined.

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MOTION OF SAND GRAINS BY WIND

Ryoma Kawamura*

1. Introduction /500**

If we go out to a beach on a windy day, we can see that the surface sand ripples under the wind. A traveller in a desert knows how annoying a sandstorm can be. Although our country (Japan) does not have any desert, there are a lot of places where invasion and erosion by sand can cause great damage and for this reason these places devote much effort in preventing it from happening. Consequently, to study the motion of sand in a wind is not only an interesting academic problem, but it would also provide the needed knowledge in establishing erosion control. In the following sections, the results of present research on this problem will be described.

2. Sand Movement Disturbed by a Wind.

Wind does not always blow in a regular pattern. It changes its direction as well as its speed. Such a wind pattern is called turbulence, and it is different from what we commonly call a wind. To study sand movement, we must first know how a sand particle moves around under the influence of turbulence. An object as small as a sand particle, when it moves in the air,

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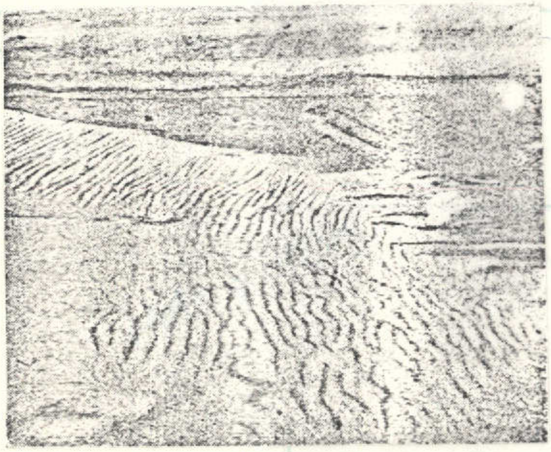

Ripple-Formation on the surface of a sand dune.

due to the small Reynolds number in such a case, has a resistance proportional to its speed according to Stokes' law. Although a sand particle generally has a very irregular shape, it can be approximated to a sphere with the equivalent volume. The motion of this sphere in the horizontal direction (the x-direction) can be worked out as follows:

$$m\ddot{x}=3\pi\mu d(u-\dot{x}) \qquad (1)$$

where m: the mass of the sand particle; μ: the coefficient of viscosity of the air; d: the diameter of the sand particle, and u: the x-component of the air velocity. If we assume the horizontal speed of the turbulence in the simple form of $u = U + u_0 \sin wt$, Equation (1) can be solved with respect to an appropriately chosen origin, namely

$$\dot{x}=U(1+Ae^{-\beta t})+u_0 B\sin(\omega t-\delta)$$

where

$$\beta=3\pi\mu d/m,\ B=1/\sqrt{1+\left(\frac{\omega}{\beta}\right)^2},\ \delta=\tan^{-1}\omega/\beta$$

and A is a constant.

If we compare the second term of the solution with the air oscillation, we see that the sand particle oscillates with the air, and its amplitude is B times the air oscillation with a phase retardation of δ. As we shall see later, when a sand particle moves around in turbulence, it takes about 0.2 sec for it to join the turbulence. Thus, this time delay ought to be taken into account to begin with. If $w = 2\pi \times 5\ \mathrm{sec}^{-1}$, then the relation between the diameter of the particle, d, and the amplitude, B, can be shown, as in Table 1; but $\mu = 1.87 \times 10^{-4}$ gm/(cm·sec) and the density of the sand particle $\delta = 2.65\ \mathrm{gm/cm^3}$. From the Table, one can see that for diameters smaller than 0.1 mm, there is the possibility for the sand particle to be captured by the turbulence, while for diameters larger than 0.25 mm, the sand particle would not be greatly influenced by turbulence. A small particle takes a long time to fall down to the ground, because air viscosity slows down its velocity. Consequently, when one considers sand particles with diameters smaller than 0.1 mm, the problem should be treated by the air diffusion coefficient, as in the case of temperature and water vapor. On the other hand, sand particles with diameters greater than 0.25 mm, not greatly influenced by turbulence, move in simple projectile motions.

TABLE 1.

$d$mm	B
0.05	0.850
0.10	0.374
0.25	0.064
0.50	0.016
1.00	0.004

3. Types of Sand Motions

Sand motions due to wind are of 3 types: suspension, saltation, and surface creep. This fact was first observed by Bagnold and Chepil. The first kind of motion — namely, suspension - is mainly for minute particles with diameters around 0.1 - 0.05 mm. The particles can be carried for a long distance, and in an ordinary wind they reach an altitude of several meters. This fact confirms what we have described in the previous section. The second kind of motion, saltation motion, is the jumping up-and-down motion by sand particles, as shown in Figure 1. This is mainly by particles with diameters of 0.2 - 0.3 cm. In this motion, sand particles jump up drawing a flying curve, then fall down to the ground to collide but bounce back up again, or to be caught by the sand surface. This motion is restricted, itself, within a height of 2-20 cm from the sand surface, and thus the flying motion distance is also much shorter than that covered by suspension motion. The third kind of motion, namely surface creep, is for sand particles gliding along the surface of the ground, and it is mainly for larger particles. If one investi-

Figure 1.

gates the sizes of sand particles on a sand dune where sand shifts around intensely and dunes come and go quickly, one finds the distribution of the sizes of sand-particles is always quite uniform - that is, the majority of the particles are in the range of 0.2 - 0.3 mm, and very few are smaller than 0.05 mm or larger than 0.5 mm. Such a distribution in fact characterizes a sand dune. And in a sand dune, one may safely assume that diameters of particles are all about 0.25 mm. Consequently, in a sand dune one does not consider suspension motion at all, but limits himself to the saltation motion.

4. Wind Intensity in Moving Sand Particles.

According to Chepil's experiments, when a wind reaches a certain speed, sand particles start to roll on the surface of a sand dune and become accelerated. After the accelerated sand particles collide with bumps on the surface, they begin to jump up to start saltation motion. In this case, the relationship between the wind speed and the diameter of a sand particle is very similar to the situation when a river carries along chunks of mud.

There have been a lot of experiments, since ancient times, to observe the motion of mud chunks in a river. The experimental results are arranged, with respect to the wind speeds of certain values, and wind speeds of interest are limited mainly to those close to the ground surface. The height z and the wind speed U have the following relationship:

$$U = 5.75\, v_* \log 30\, z/k, \quad v_* \div \sqrt{\tau_0/\rho} \qquad (2)$$

where τ_o: shear force which moves sand particles, ρ: density of the air, and k: the degree of coarseness of the sand surface. The coarseness k of the surface varies from place to place, but on a flat surface, it is about equivalent to the diameters of particles, d. The quantity v_* is called the frictional velocity, because it has the dimension of a velocity, but really v_* is a parameter to be fixed by the experiment of the moment when sand particles start to move. Figure 2 is due to Chepil, and v_{*t} in the figure is the frictional velocity obtained when sand particles started to move. According to the figure, within the limit of $\sigma d>0.1$, v_{*t} is proportional to $\sqrt{d}$. However, in many other experiments, the relationship between v_{*t} and d is shown by the following formula:

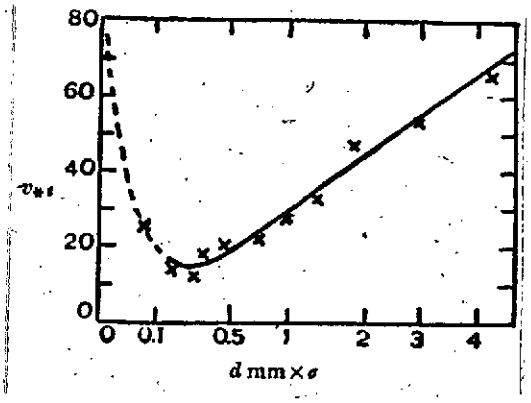

Figure 2.

$$v_{*t}=A\sqrt{\frac{\sigma-\rho}{\rho}gd} \qquad (3)$$

where σ: the density of sand particles and A: constant. This empirical formula in fact can be theoretically worked out as follows: In the vicinity of the surface, there are N sand particles in a unit volume, then the volume of a sand particle is $N\cdot\pi d^3/6$. If we call this volume λ, then $N = 6\lambda/\pi d^3$. The total number of sand particles in a unit surface of the sand which is being directly bombarded by the wind is called n; then

$$n=N^{\frac{2}{3}}=\left(\frac{6\lambda}{\pi}\right)^{\frac{2}{3}}\frac{1}{d^2}$$

On the other hand, the shear force on a unit surface is ρv_*^2, and this force is equal to the resistance of n particles on a bump of the sand surface. Consequently, the resistance D applied on one sand particle is shown in average as follows:

$$D=\frac{\rho v_*^2}{n}=\rho\left(\frac{\pi}{6\lambda}\right)^{\frac{2}{3}}d^2 v_*^2$$

Now if the static friction angle is φ_0, then the condition to move a particle is $D\geq mg\tan\varphi_0$. If v_{*t} is the friction velocity when all the sand particles of a surface are moving in a statistical sense, then

$$\frac{\rho}{n}\cdot\left(\frac{\pi}{6\lambda}\right)^{\frac{2}{3}}d^2 v_{*t}^2=mg\tan\varphi_0$$

With $m = \pi\sigma d^3/6$, one finds

$$v_{*t}=\pi^{\frac{1}{6}}\cdot(6\lambda)^{\frac{1}{3}}(\tan\varphi_0)^{\frac{1}{2}}\sqrt{\frac{\sigma}{\rho}gd} \qquad (4)$$

In taking into account the air buoyancy, one gets Formula (3). Actually, $\varphi_0 \doteqdot 30°$ and $\lambda \doteqdot 0.5$. These values can be inserted in (4) but they will yield a value of v_{*t} which is nearly 10 times larger than the experimental value. This is due to the fact that this formula is obtained with the assumption as given in (4) that all particles move around in the same manner. However, in reality many particles are so small that they need a far smaller value of v_{*t} to move them. Consequently, it is natural that the theoretical value of v_{*t} from (4) is much larger than the real value. There is another theoretical approach in which it is assumed that v_{*t} is entirely due to the force which the wind exerts on the sand surface. However, we believe this approach does not have any theoretical justification. In Figure 2, in the region of σd<0.1 and for a smaller d, v_{*t} increases. According to Chepil's explanation, when particles are in fine granules, the surface has a small coarseness and, consequently, the situation is equivalent to the case of smooth ground, on which a wind blows. Hence, no turbulence can reach the vicinity of the surface, but a low-layer of a gradient flow is formed and sand particles can bury themselves in this layer. Consequently, for the same v_* value, the wind velocity at the sand surface - namely, at z=d — becomes smaller, for a smaller d. Since the resistance which moves sand particles is proportional to the surface wind velocity, for a smaller d one needs a larger v_{*t}. Although v_{*t} can be worked out theoretically for d in this region, the effort may be omitted here, because fine granular sand particles are not a part of the problem related to the motion of a sand dune.

5. Stationary Sand Flows

In the previous section, we have described the condition with which sand particles can be moved by the resistance applied to the sand, and the resistance is determined by the wind velocity in the vicinity of the sand, namely, in the region of z=d. /502

If we call this velocity V, then when V reaches a certain value, unstable sand particles on the surface x will start to roll, and then be accelerated by the wind. When the accelerated particles hit a bump on the surface, they jump up into the air. The particles will come back down due to the gravity. However, if particles on the way down do not lose too much energy to the wind, their impact to the ground can create more unstable sand particles. These unstable particles can again be activated by the wind. The surface wind velocity V can move all the unstable particles, and thus the amount of flying sand particles will increase, but since flying sand particles have to obtain their momenta from the wind, the wind eventually will be slowed down too. When V goes down below the capacity to move x sand particles, the flying sand particles will all go down to the ground and the amount will decline. Thus, the wind velocity and the amount of flying sand regulate each other to produce a stationary state. A stationary sand flow indicates that the surface wind velocity is always strong enough to move sand particles. When the wind velocity is at a certain value, it creates the largest amount of sand particles (called the saturated amount of sand particles). The horizontal shear force of the air is equal to the sum of the shear stress τ_s due to the moving sand particles and the Reynolds' stress τ_w due to the turbulence: namely, $\tau=\tau_s+\tau_w$. In the vicinity of the ground surface, one can neglect the horizontal pressure gradient, and therefore τ is constant in the vertical direction and equals the shear stress τ_o applied to the moving sand particles. When the amount of flying sand increases, so does τ_s. However, since τ_s can never exceed τ_o, the condition for saturation is therefore $\tau_s=\tau_o$. At a source of resonably dry sand, the amount of flying sand is always at the saturation point except at the periphery of the source.

6. The Motion of Flying Sand in the Air.

As mentioned before, for particles of diameters around 0.25mm, one does not have to consider turbulence. Although the wind velocity V varies in the vertical direction, yet since we cannot handle this variation, the wind velocity V has to be assumed as constant. If the wind direction is taken as the x-direction, and the vertical direction is the z-direction, then the equations of motion can be written as follows, assuming that the resistance force is proportional to the velocity:

$$m\ddot{x}=\alpha(V-\dot{x}) \tag{5}$$

(α is a constant)

$$m\ddot{z}=-mg-\alpha\dot{z} \tag{6}$$

Now at $t=0$; $x=0$, $z=0$, $\dot{x}=u_1$ and $\dot{z}=w_1$, and for a short jumping distance, u_1 and w_1 can be assumed as very small. Then

$$\left.\begin{aligned} x&=u_1+\frac{1}{2}(V-u_1)\beta t \\ \dot{x}&=u=u_1+(V-u_1)\beta t \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} z&=\left(w_1-\frac{g}{2}t\right)t \\ \dot{z}&=w=w_1-gt \end{aligned}\right\} \tag{8}$$

where $\beta=\alpha/m$. Consequently, the falling velocities, u_2 and w_2, jumping time T, jumping distance L and jumping height h of a sand particle can be found to be

$$\left\{\begin{aligned} &u_2=u_1+2\frac{\beta V}{g}w_1,\ w_2=-w_1,\ T=\frac{2w_1}{g}, \\ &L=\frac{w_1}{g}(u_1+u_2),\quad h=\frac{w_1^2}{2g}. \end{aligned}\right. \tag{9}$$

The resistance of the particle is, according to Stokes' law, found from $\alpha=3\pi\mu d$, but since the Reynolds number 10 - 50 obtained by comparing with the experimental results, $\alpha=2\times3\pi\mu d$.

7. Distribution of Flying Sand in the Vertical Direction

In stationary sand flow, the number of sand particles jumping off the ground must be equal to the number of sand particles falling down to the ground. This number is called n_0.

According to the experiments by Bagnold and Chepil, since most of the sand particles are jumping up vertically, in (9), u_1 can be neglected, as compared to w_1 and u_2. Since the situation at the sand surface is totally random, the velocity w_1 distribution of jumping particles follows the Maxwell distribution. However, the probability to move from $w_1<0$ to $w_1 \geqq 0$ is zero. If $g(w_1)$ is the probability $g(w_1)dw_1$ for a particle to fall at a velocity w_1 between w_1 and w_1+dw_1, then

$$g(w_1)\,dw_1=\frac{1}{\pi w_0}\exp\left[-\frac{1}{\pi}\frac{w_1^2}{w_0^2}\right]dw_1,\quad w_1\geqq 0$$

where w_o is the average value of w_1. From (9), one can replace w_1 by h, then the probability $f(h)dh$ for a particle to fall between h and $h+dh$ is

$$f(h)=\frac{1}{\pi\sqrt{h_0}}\frac{1}{\sqrt{h}}\exp\left[-\frac{1}{\pi}\frac{h}{h_0}\right] \qquad (10)$$

where h_o is the average of h. Next, if $n(z)dz$ is the number of sand particles at the height in the range between z and $z+dz$ and the average x-component of the velocity is $\bar{u}$ at this height, then in a unit time, the number of sand particles flow through this height is $q(z)\,dz$ which can be shown as

$$q(z)\,dz=m\cdot n(z)\bar{u}(z)\,dz \qquad (11)$$

Now n and $\bar{u}$ can be found as follows. If particles stay in the range of z and $z+dz$ for the duration of dt, then it is dz/w. From (7) and (8), $w^2=2g(h-z)$ and consequently,

$$dt=\frac{dz}{\sqrt{2g\,(h-z)}}$$

Since it takes the same amount of time for a particle to fall down as it flies up, the time for a particle to finish one jump is 2 dt. During this time, a total of $n_o dt$ makes the jumps. Consequently,

$$\begin{aligned} n(z)\,dz &= \int_{h=z}^{\infty} n_0 f(h)\,dh\cdot 2dt \\ &= n_0\cdot\frac{dz}{\pi}\sqrt{\frac{2}{gh_0}}\int_z^{\infty}\frac{\exp\left[-\frac{1}{\pi}\frac{h}{h_0}\right]}{\sqrt{h(h-z)}}\,dh \end{aligned} \qquad (12)$$

Now from (7) and (8), the x-component of the particle-velocity at z, /503

$$u=\frac{\beta V}{g}\left[\sqrt{2gh}\mp\sqrt{2g(h-z)}\right]$$

The ($\pm$) sign shows the direction of jumping up or falling down, respectively. Consequently, in the up-and-down motion, the average value of u is independent of z but depends on h alone, that is if we call this average velocity $\bar{u}(h)$, then

$$\bar{u}(h) = \frac{\beta V}{g} \sqrt{2gh}$$

Finally,

$$q(z) = \int_z^{\infty} m n_0 f(h) dh \cdot \frac{2}{\sqrt{2g(h-z)}} \cdot \bar{u}(h) = 2 m n_0 \cdot \frac{\beta V}{g} \exp\left[-\frac{1}{\pi} \frac{z}{h_0}\right] \qquad (13)$$

This shows that z and log q(z) have a linear relationship. Figure 3 shows the experimental values obtained by Chepil, and one sees that the theoretical formula generally satisfies the curve. The above formula was obtained by assuming w_1 and, therefore, h are negligible, and V is assumed to be at a constant. For a larger h, one needs a large V and therefore, the average velocity of sand particles must be also large. Consequently, for a large z value, the amount of sand particles ought to be larger than that obtained from the above formula. This is why in the experimental curve of Figure 3, the curve becomes a straight line for large z-values.

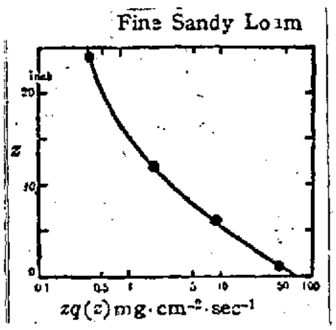

Figure 3.
(in a logarithmic scale)

8. <u>Relationship Between Wind Intensity and the Amount of Flying Sand</u>

We divide the air space into 2 layers. The top layer has no influence on the flying sand, while in the bottom layer the effect on the shear force of turbulence is assumed to be absolutely negligible. This lower layer of air space is called the layer of flying sand. In the flying sand layer, the wind loses its momentum to sand. This means that flying sand particles are like grass in a grassland bending to a wind and thus this layer can also be understood as a surface with coarseness. The coarseness, of course, depends on the thickness h_s of the flying sand layer. The velocity distribution of the layer can be shown as (2)

$$U = 5.75\, v_* \log \frac{30z}{ch_s},$$

where c is a constant.

Consequently, immediately outside the layer - namely, when $z=h_s$ - the wind velocity is proportional to v_*. Since the shear stress is ρv_*^2, work done in a unit time outside the layer is thus proportional to ρv_*^3. This work is provided to the layer in order to increase the kinetic energy of the flying sand. Now we bring average values into the problem, taking the average values of jumping-up velocity and the horizontal component of the falling-down velocity of a sand particle as w_1 and u_2, respectively. Also we ignore the error in assuming that the average of the square is equal to the square of the average. Then the above working relationship becomes

$$\frac{1}{2} m n_0 u_2^2 = k \rho v_*^3, \quad \text{k is a constant} \qquad (14)$$

Now since the momentum increase in one jump is mu_2 (where u_1 is neglected because it is a small value) and in a state of saturation the shear stress applied to the sand surface is equivalent to the momentum change of a sand particle,

$$m n_0 u_2 = \rho v_*^2. \qquad (15)$$

And if one neglects u_1 in (9),

$$u_2 = 2 \frac{\beta V}{g} w_1 \qquad (16)$$

Consequently, from these 3 equations (14), (15) and (16), one can find u_2, w_1 and n_0 and they are

$$u_2 = 2k v_*, \quad w_1 = k \frac{g}{\beta V} v_*, \qquad (17)$$
$$n_0 = \frac{1}{2k} \frac{\rho}{m} v_*.$$

Hence, the average values of a jumping distance and the height, L and h, are from (9) as follows:

$$L = 2k^2 \frac{1}{\beta V} v_*^2, \quad h = \frac{k^2}{2} \frac{g}{\beta^2 V^2} v_*^2 \qquad (18)$$

Next, we consider an infinite vertical plane, perpendicular to the direction of the wind. The amount of sand particles passing through a unit area on this plane in a unit time is Q and

$$Q = mn_0L = k\frac{g}{\beta V}\cdot\frac{\rho}{g}v_*^3 \qquad (19)$$

According to the experiments by Bagnold, a sand particle of a diameter around 0.25 mm has V ≑ 250 cm/sec and Q ≑ $1.5\rho v_*^3/g$. Other constants can also be shown to be $m=2.17\times10^{-5}$g, $\beta=4.00$ sec^{-1}, and $\rho=1.25\times10^{-3}$g/cm^3. If one inserts these values into (17), (18) and (19), then

$$\begin{cases} k=1.53, \ u_2=3.06\,v_*, \ w_1=1.50\,v_*, \\ n_0=18.8\,v_*, \ L=4.59\,v_*^2/g, \ h=1.12\,v_*^2/g. \end{cases} \qquad (20)$$

TABLE 2.

v_* cm·sec^{-1}	u_2 cm·sec^{-1}	w_1 cm·sec^{-1}	n_0 sec^{-1}·cm^{-2}	L cm	h cm	Q gm·cm^{-1}·sec^{-1}
20	61.2	30	376	1.6	0.45	0.015
40	122	60	752	7.5	1.83	0.122
60	183	90	1130	17.3	4.11	0.414
80	245	120	1500	30.0	5.60	0.931

Thus for each v_* value, the other quantities can also be shown as in Table 2. Consequently, the amount of flying sand Q is generally proportional to the third power of the wind velocity ($v*^3$) for a certain given height. It is also shown in the formula that a weak wind for a longer duration moves the sand more effectively than a strong wind blowing for a shorter period of time.

9. Surface Creep /504

We consider a jumping motion in which sand particles collide onto the sand surface and bounce up. If the x- and z-components of the velocity before and after the collision are (u_2, w_2) and (u_1, w_1), respectively, then a sand particle will receive in the x-direction a force of $m(u_2-u_1)$ and in the z-direction - a force of $m(w_2-w_1)$. Consequently, when some condition is satisfied in the collision, the particle will slide, making a "surface creep" motion:

$$m(u_2-u_1) \geqq [m(w_1-w_2)+mg]\tan\varphi$$

where φ is the static frictional angle of the sand particle. We can rewrite the condition as follows:

$$\frac{u_2-u_1}{\tan\varphi}-(w_1-w_2)\geqq 0$$

From (17) of the previous section, one knows that the left side of the condition on the average is proportional to v_*. If in a unit time and in a unit volume, the number of collisions is called n_o, then the above condition for n_o collisions in a unit time and in a unit volume is proportional to $n_o v_*$. Next, if a sand particle slides on for a distance of L_o, in a collision the particle will do a work of L_o x mg tan φ' (φ' to the sand surface) and this amount of work must be porportional to the loss of the kinetic energy of the particle. Taking the proportional constant as C_1, one gets

$$L_o mg\tan\varphi'=C_1\frac{1}{2}m[(u_2^2-u_1^2)+(w_2^2-w_1^2)]$$

As we did before, the right side is proportional to mv_*^2 and the average value of L_o is proportional to v_*^2/g. Consequently, if the amount of sand particles doing surface creep on a unit area is called Q_c, then

$$Q_c\sim m\cdot v_* n_o\cdot\frac{v_*^2}{g}$$

By use of (17) and replacing n_o,

$$Q_c=C\cdot\frac{\rho}{g}v_*^3,\ C \text{ is a constant.} \tag{21}$$

By comparing (19) and (21), one gets Q_c/Q=constant. Bagnold made some experiments in deserts, and he found that for the average diameter of 0.25 mm, the ratio Q_c/Q is independent of the wind velocity but is always about 0.25. The observation agrees with the above theory.

10. Appearance of Sand-Ripples

If we go to a beach on a very windy day, we can see sand waves with wavelengths of several cm, moving slowly along with the wind. Such sand waves are called sand-ripples. Up to the present time, various observations and researches have been made on these sand-ripples. Exner tried to explain the formation of sand-ripples as due to some turbulence, but his theory fails to produce the wavelength of a sand-ripple, and it also does not show the relationship between the wind intensity and the ripple formation. Since ordinary sand particles, as described above, are not greatly influenced by turbulence, due to this point alone, one can argue that Exner's approach is questionable. According to Chepil's experiments, for sand particles with diameters either smaller than 0.05 mm or larger than 0.7 mm, no ripple can be formed by a wind with intensities of any magnitude. Also Bagnold reported a strong relationship existing between the wavelength of a sand-ripple and the wind intensity. Based on all these points, it is appropriate to assume that a sand-ripple formation is theoretically related to the problem of the saltation motion of sand particles. We can study the stability problem on the sand-surface, based on this viewpoint. We first assume that a wind is blowing over a flat surface with a uniform velocity. The air viscosity is neglected. If the wind receives a velocity disruption of the amount of u and w in the x and z-direction, respectively, and the air pressure is taken as p, then the equation of motion and equation of continuity can be shown as

$$\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x},$$
$$\frac{\partial w}{\partial t}+V\frac{\partial w}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x},$$
$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0.$$

Now we express the unevenness of the sand-surface by the height of the sand, y, then the slope

$$\frac{\partial y}{\partial x}=\left(\frac{w}{V}\right)_{z=0}$$

With $u=u_o\exp(\sigma t+i\nu x)$ and $w=w_o\exp(\sigma t+i\nu x)$, one gets the following solutions:

$$\begin{cases} u=u_0\exp[\sigma t+i\nu x-\nu z], \\ w=iu_0\exp[\sigma t+i\nu x-\nu z] \\ y=\frac{u_0}{V}\frac{1}{\nu}\exp[\sigma t+i\nu x] \end{cases} \qquad (22)$$

The time variation of the sand height (y) at a certain spot is proportional to the difference between the number of sand particles falling down on this spot and the number of sand particles jumping off the spot. Since most of the sand particles take a similar path in a saltation motion, particles jumping off from the spot where x lies between ξ and $\xi+d\xi$ will fall in x and x+dx. Consequently,

$$\Delta y \cdot \Delta x = A[N(\xi)\Delta\xi - N(x)\Delta x]\Delta t.$$

where A is a positive constant, and N is the number of particles jumping off from a unit surface in a unit time. By rearranging the above relation, one gets

$$\frac{\partial y}{\partial t} = A\left[N(\xi)\frac{d\xi}{dx} - N(x)\right] \tag{23}$$

If sand particles, due to the wind dusturbance, jump up with a velocity of w_1, and if one assumes that a strong wind velocity will provide a strong jumping velocity w_1, then

$w_1 = \overline{W}_0 + au_{z=0}$; a is a positive constant,

where the second order term is neglected. $\overline{W}_0$ is the jumping velocity when no disturbance is present. One can find the jumping distance from (9) as

$$L = L_0 + bu_{z=0}, \tag{24}$$

where b is a positive constant, and L_0 is the distance when no disturbance is present. Consequently,

$$L(\xi) = x - \xi,\ L(\xi+\Delta\xi) = x + \Delta x) - (\xi + \Delta\xi)$$

and therefore

$$\frac{d\xi}{dx} = \left(1 + \frac{dL(\xi)}{d\xi}\right)^{-1} \tag{25}$$

From (22), (24), and (25),

$$\frac{d\xi}{dx} = 1 - ibu_0\nu\exp[\sigma t + i\nu(x - L_0)] \tag{26}$$ /505

Now assuming that the total number of the particles involved in the saltation motion is constant, namely $N(\xi) = N(x) = N_0$, from (22), (23) and (26) one gets

$$\frac{1}{V\nu}\sigma = -iAN_0 b\nu\exp[-i\nu L_0] \tag{27}$$

(27)* is the equation which determines the stability of a sand

*Translator's Note: Mistakenly printed as (2) in the original foreign text.

surface. When the real part of σ is positive, the situation is unstable, and when it is negative the situation is stable. If one sets

$$\sigma=\sigma_1+i\sigma_2 \quad (\sigma_1 \text{ and } \sigma_2 \text{ are real numbers})$$

then (27) becomes

$$\begin{cases} \sigma_1 = -AbVN_0\nu^2 \sin \nu L_0 \\ \sigma_2 = AbVN_0\nu^2 \cos \cdot L_0 \end{cases} \qquad (28)$$

Consequently, $\sigma_1>0$ indicates the region of instability, namely $(2m+1)\pi>\nu L_0>2m\pi$ (m positive integer). In this region, the most unstable point of σ_1 is at

$$-2 \tan \nu L_0 \qquad (29)$$

The larger is m, the larger the value of σ_1 will become, and therefore, a greater instability appears. Then the question remains of when and what value of m creates a sand-ripple. The wavelength of a sand-ripple is obtained as the roots λ_s of (29), namely $\nu L_0=\theta m$, or

$$\lambda_S=\frac{2\pi}{\nu}=\frac{2\pi}{\theta m}L_0$$

It is known from (18) that L_0 is proportional to v_*^2. λ_s is also proportional to v^{*2} and consequently Figure 4 is due to Bagnold from his experiments and it shows the experimental relationship between λ and v_*. One sees that the theory agrees quite well with the experimental data. Actually, a sand ripple appears when m=o.

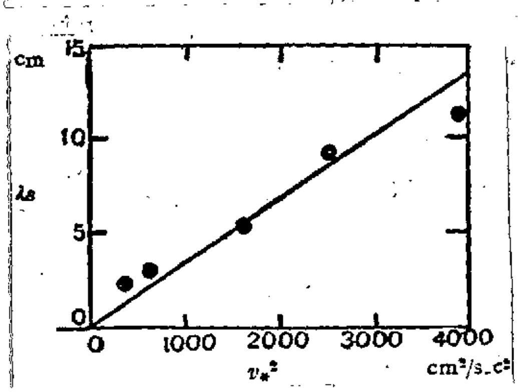

Figure 4.

A sand-ripple appears when νL_0 satisfies (29), and when that happens, $\sigma \doteq 0$, that is, the propagation speed of the wave is close to zero. This is due to the fact that we have so far only considered the saltation part of the motion. In reality, the surface creep motion will push the sand-ripple slowly forward.

In the above theoretical explanation, many assumptions and omissions have been made. Consequently, the theory is far from complete. We hope to be able to fill in these imperfections in a future study.

11. Peculiarities in the Motion of Sand and a Sand-Hill

The contents of this section is not original research of the present author. However, since there are some interesting peculiarities in the movement of sand, we want to include it in this article, along with some simple explanation of the problem of a sand dune.

We have noted above that sand can be sorted out in sizes by a wind. In an ordinary sand source, all knids of sizes are usually present. Suppose we mix particles of different sizes artificially into some sand and blow a wind over to observe the distribution of particle sizes afterwards, both in the wind and on the ground. We find the distribution on the ground is different, whether we have mixed the sand with alot of fine granules or a lot of larger particles, but in the wind the distribution does not change due to the artificial mixing. This is due to the sorting effect of the wind, and the unchangable distribution of sand particles in the wind is the same distribution one may find in a sand dune. Fine granules are very difficult to be picked up by a wind, as shown in Figure 2, because they are protected by large particles. On the other hand, large particles are obviously difficult to be blown up by a wind. In effect, only particles with diameters around 0.25 mm can easily be picked up by a wind. Consequently, around a sand dune, one finds the average diameter of sand particles is around this value and the distribution has a quite definite pattern. Now we should consider the aforementioned saturated amount of sand particles; in other words, we should consider how for a given wind intensity, the condition of a sand surface affects the saturated amount. If the ground has accumulated a lot of large sand particles or small rocks, sand particles, after they collide with the ground, bounce back more strongly, and consequently, they can reach a higher altitude with a larger vvelocity. Hence, for the same intensity of wind, a larger amount of sand particles will be blown up. This situation is similar to the case of a solid ground. Contrary to this, in the case of a river surface, the amount of flying sand is zero because sand particles will all sink to the bottom of the river. In a large sand source, such as a desert, there are a large amount of sand particles with large diameters, and in such a place smaller sand particles do not accumulate but they are carried away by winds. Consequently, only coarse sand particles are eventually left behind. Sand particles carried away by winds will accumulate at some sand dune area. This is one of the reasons a sand dune is formed.

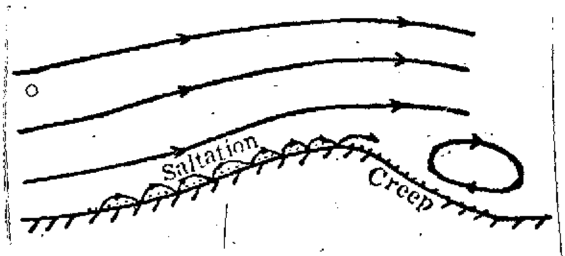

Figure 5.

Next, a few simple explanations are needed for the sand dune problem. A complete reason for sand dune formation is not yet attainable, as far as the present author knows. The cross-section of a sand dune, formed naturally, can be shown in Figure 5. The angle of stability of sand is found to be at 8-15° for an upward /506 wind and it is almost 30° for a downward wind. In observing the distribution of sand particles, one finds that particles at the foot of a downward hill are coarser than those at the upward hill. This is due to the fact that smaller particles, which can perform saltation motion, roll upwards but do not fall down to the foot of a hill to remain static, while the coarser particles which can perform only surface-creep motions accumulate slowly at the foot of a hill. Exner has made some simple calculation on the movement of sand dunes, and we want to introduce it here.

Suppose a sand dune moves a distance of Δl in a time Δt in the direction of the downward wind, preserving its original shape (Figure 6). If M is the amount of sand being moved by the wind in a unit area in a unit time and ρ_s is the density of the sand dune (not the density of a sand particle), and h is the height of the dune, then

Figure 6.

$$\Delta l/\Delta t = M/\rho_s h$$

where it has been assumed that the upward angle is smaller. The magnitude of ρ_s has been found elsewhere to be 1.22 gm/cm^3. This theory due to Exner shows that the speed of sand dune movement is proportional to M, but inversely proportional to the height of the dune. Consequently, a higher sand dune moves more slowly. To prevent sand intrusion, one of the methods is to make the artificial sand dune high. The above theoretical formula indeed agrees well with reality. For a sand dune about 50 meters high, the moving distance in one year is about 10 m, although it varies somehow, depending on the intensities of the wind blowing during the year.

In the region of sand dunes, there are all kinds of sand-ripples with wavelengths ranging from small to as large as 100 m. There are also many sand dunes which fit into the aforementioned standard corss-section, so-called Barchan. However, owing to space limitations, this discussion will be omitted in this article.

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